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LETTER TO THE EDITOR

New non-Fermi-liquid-type behaviour by a two-band system in normal phase

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Abstract

We are reporting a new non-Fermi-liquid-type normal phase that has a well defined Fermi energy, but without showing any non-regularity in the momentum distribution function n_k in the whole momentum space, the sharp Fermi momentum concept being undefinable. The system contains a natural built-in gap that is visible in the physical properties of the system at nonzero temperatures. The presence of a flat band in multi-band interacting Fermi systems with more than half filling is the key feature leading to such a ground state, which is not restricted to one spatial dimension and emerges in the proximity of an insulating phase.

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Our understanding of the behaviour of interacting fermionic many-body systems is intimately connected to the concept of a Fermi liquid introduced by Landau many decades ago [1]. In a normal state that preserves all symmetry properties of the high-temperature phase of a fermionic system, the Fermi liquid behaviour has been clearly observed in the normal state of He-3 and simple metals [2]. It has the meaning that, in spite of the inter-particle interactions, the low-energy behaviour of the system can be well described within a picture of weakly interacting quasi-particles [3]. This picture can also be mathematically formulated [4]. In these terms, in a normal Fermi liquid, the following hold. (a) There is a one-to-one correspondence between the non-interacting one-particle states and interacting single-particle states. This is concretely obtained by describing the interacting system using a perturbation theory that is convergent up to infinite order. (b) The single-particle Green functions have a quasi-particle pole that gives rise to a discontinuity of the momentum distribution function n_k at the Fermi surface whose position is specified by a sharp Fermi momentum value $\vec{k}_{\mathrm{F}}$. (c) The residual quasi-particle interactions can be described by a small number of parameters, called Landau parameters, which can be deduced from a microscopic theory taking into account non-divergent two-particle vertex functions [5]. In the last decade, however, non-Fermi-liquid behaviour has been observed experimentally in the normal phase of a variety of materials, including higher-than-one-dimensional systems of great interest. Examples are

high-temperature superconductors [6], heavy fermions [7], layered systems [8], quasi-one-dimensional conductors, doped semiconductors, systems with impurities, materials presenting proximity to the metal–insulator transition [3] etc. These results are often discussed in terms of multi-band models [9], the presence of some kind of gap in the normal phase being clearly established in many cases and the subject of intensive studies [9,10]. The last decade witnessed a huge intellectual effort [11] for the understanding of the non-Fermi-liquid behaviour in the normal phase [12] of fermionic systems. On the theoretical side however, for pure systems, the existence of a non-Fermi liquid in a normal phase has been exactly proved only for the one-dimensional case (i.e. Luttinger liquid [13]). So far, the possibility of extending the proof to two spatial dimensions has not been demonstrated rigorously. In fact, a rigorous theory of a non-Fermi-liquid normal state in more than one spatial dimension is missing. For this reason, the theoretical understanding of different phenomena observed in the materials listed above is relatively poor and theoretical advance in this subject is badly needed. Our work on the periodic Anderson model (PAM) at nonzero and finite on-site Coulomb repulsion (U) (a prototype of two-band systems containing strong correlation effects) was motivated by this state of affairs. We present in this Letter, for the first time, an exact solution for this model. The solution present in a restricted (but continuous and infinite) domain of the phase diagram represents a new type of non-Fermi-liquid behaviour in a normal phase for a system that has a built-in gap. The obtained ground-state energy cannot be expressed as a sum of contributions of the on-site Hamiltonian terms, and the ground-state expectation value of the kinetic energy terms is nonzero and negative[1]. The state emerges in the vicinity of a Mott insulating phase in a continuous domain of concentration above three-quarters filling and has a well defined Fermi energy, but the Fermi momentum cannot be defined, n_k being without any non-regularity in momentum space. The property is due to the emergence of a flat band in a multi-band system with more than half filling and can be extended in an exact manner to two spatial dimensions [14]. Such features have been observed experimentally, for example in ARPES data, which, even for high-T_c materials, often reflect main bands without any sharp characteristics in n_k [15] or necessitate the assumption of the presence of flat bands [16]. Band structure calculations for layered systems often show a Fermi level positioned exactly at the bottom of a conduction band with a relatively large effective mass around its minimum, below which a gap is present [8]. Connections between the emergence of superconductivity and flat dispersions were also clearly pointed out in [17]. Flat-band features are present in heavy-fermion systems as well [18], and can even be produced by squashing carbon nanotubes [19].

We consider two bands denoted by $b = c, f$, the starting 1D Hamiltonian being $\hat{H} = \hat{H}_0 + \hat{U}$. The Hubbard term is $\hat{U} = U \sum_i \hat{n}^{\mathrm{f}}_{i,\uparrow} \hat{n}^{\mathrm{f}}_{i,\downarrow}$ and we have $\hat{H}_0 = \hat{T}_{\mathrm{c}} + \hat{T}_{\mathrm{f}} + \hat{E}_{\mathrm{f}} + \hat{H}_{\mathrm{h}}$ with the kinetic energies $\hat{T}_{\mathrm{b}} = t_{\mathrm{b}} \sum_{i,\sigma} \left[b^{\dagger}_{i,\sigma} b_{i+1,\sigma} + \mathrm{h.c.} \right]$, the on-site f level energy $\hat{E}_{\mathrm{f}} = E_{\mathrm{f}} \sum_{i,\sigma} \hat{n}^{\mathrm{f}}_{i,\sigma}$, hybridization energy $\hat{H}_{\mathrm{h}} = \hat{V}_0 + \hat{V}_1$ and

$$\begin{aligned} \hat{V}_0 &= \sum_{i,\sigma} \left[\tilde{V}_0 \, c^{\dagger}_{i,\sigma} \, f_{i,\sigma} + \mathrm{h.c.} \right] \\ \hat{V}_1 &= \sum_{i,\sigma} \left[\tilde{V}_1 \left(c^{\dagger}_{i,\sigma} \, f_{i+1,\sigma} + f^{\dagger}_{i,\sigma} \, c_{i+1,\sigma} \right) + \mathrm{h.c.} \right]. \end{aligned} \tag{1}$$

We present the case when $\hat{H}_{\mathrm{h}}$ contains imaginary coupling constants, i.e. $\tilde{V}_0 = \mathrm{i}\, V_0$ and $\tilde{V}_1 = \mathrm{i}\, V_1$, where V_0, V_1 are real. For the starting point we consider three-quarters filling. In this Letter we shall give an exact solution of this problem, valid in a restricted domain of the phase diagram relevant for our study. In order to have a clear image of the solution and its physical meaning, let us consider the $U = 0$ case first. To this end, we would like to express

[1] As a consequence, the system is not insulating.

$\hat{H}_0$ through a $\hat{B} = \sum_{i,\sigma} B^{\dagger}_{i,\sigma} B_{i,\sigma}$ term, where $B_{i,\sigma} = \alpha\, c_{i,\sigma} + \beta\, c_{i+1,\sigma} + \gamma\, f_{i,\sigma} + \delta\, f_{i+1,\sigma}$ and $\alpha,\ \beta,\ \gamma,\ \delta$ are constants to be determined. Introducing the total particle number operator $\hat{N} = \sum_{i,\sigma,\mathrm{b}} \hat{n}^{\mathrm{b}}_{i,\sigma}$, we find then that $\hat{H}_0 = -\hat{B} + \eta\,\hat{N}$ holds, if the following relations are satisfied:

$$\eta = |\alpha|^2 + |\beta|^2 \tag{2}$$

$$E_{\mathrm{f}} = \eta - \left(|\gamma|^2 + |\delta|^2\right) \tag{3}$$

$$-t_{\mathrm{c}} = \alpha^* \beta = \beta^* \alpha \tag{4}$$

$$-t_{\mathrm{f}} = \gamma^* \delta = \delta^* \gamma \tag{5}$$

$$\mathrm{i}\, V_0 = \alpha\, \gamma^* + \delta^* \beta = -(\alpha^* \gamma + \delta\, \beta^*) \tag{6}$$

$$\mathrm{i}\, V_1 = \alpha\, \delta^* = \gamma\, \beta^* = -\alpha^* \delta = -\gamma^* \beta. \tag{7}$$

Equations (4)–(7) determine through $\alpha,\ \beta,\ \gamma,\ \delta$ the value of E_{f} and η for which the presented structure of $\hat{H}_0$ is valid. With $\eta = 2\sqrt{\bar{v}^2 + t_{\mathrm{c}}^2}$, $\bar{v} = V_0\, V_1/(2\, t_{\mathrm{f}})$ we obtain

$$E_{\mathrm{f}} = \eta\,(1 - t_{\mathrm{f}}^2\, V_1^{-2}) \qquad V_1^2 = -t_{\mathrm{c}}\, t_{\mathrm{f}}. \tag{8}$$

Taking now into consideration that $B^{\dagger}_{i,\sigma} B_{i,\sigma} + B_{i,\sigma} B^{\dagger}_{i,\sigma} = |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2$ and imposing the condition $\langle \hat{N} \rangle = 3\, L$ where L represents the number of lattice sites (three-quarters filling), we find $\hat{H}_0 = E_{\mathrm{g}} + \hat{P}$, where $E_{\mathrm{g}} = L\,\eta\,\left(1 - 2\,|m|^2\right)$ and $m = t_{\mathrm{f}}/\tilde{V}_1$ has been introduced. The operator $\hat{P} = \sum_{i,\sigma} B_{i,\sigma}\, B^{\dagger}_{i,\sigma}$ being positive semidefinite, E_{g} is the ground-state energy, and the ground-state wavefunction is that $|\psi_{\mathrm{g}}\rangle$ for which we have $\hat{P}\,|\psi_{\mathrm{g}}\rangle = 0$. We demonstrate now that

$$|\psi_{\mathrm{g}}\rangle = \prod_{i=1}^{L} \left(\prod_{\alpha=1}^{3} F_i^{(\alpha)} \right) |0\rangle \tag{9}$$

where $|0\rangle$ is the bare vacuum with no fermions present, $D_{i,\sigma} = \alpha^* \left(c^{\dagger}_{i,\sigma} + m\, f^{\dagger}_{i,\sigma}\right) + \beta^* \left(c^{\dagger}_{i+1,\sigma} + m^*\, f^{\dagger}_{i+1,\sigma}\right)$ and $F_i^{(\alpha=1,2)} = D_{i,\sigma(\alpha)=\uparrow,\downarrow}$. In order to see this, one can easily verify that $B^{\dagger}_{j,\sigma}\, F_i^{(\alpha)} = -F_i^{(\alpha)}\, B^{\dagger}_{j,\sigma}$ independent of the indices, and $B^{\dagger}_{i,\sigma}\, D_{i,\sigma} = 0$, so $\hat{P}\,|\psi_{\mathrm{g}}\rangle = 0$, and $|\psi_{\mathrm{g}}\rangle$ is the ground state. We must stress that for $U = 0$ the ground state is entirely given by $F_i^{(1)}$ and $F_i^{(2)}$, the $F_i^{(3)}$ operator being completely arbitrary apart from the requirement that it introduces under the $\prod_i$ product L electrons into the system. The concrete expression of $F_i^{(3)}$ is fixed by the nonzero U as follows. When $U \neq 0$, the Hamiltonian contains, besides $\hat{H}_0$, the Hubbard $\hat{U}$ as well. However, one may observe that $\hat{U}$ can be exactly transformed as $\hat{U} = U\,\hat{P}' + U\,\sum_{i,\sigma} \hat{n}^{\mathrm{f}}_{i,\sigma} - U\,L$ where $\hat{P}' = \sum_i \hat{P}'_i$ and $\hat{P}'_i = (1 - \hat{n}^{\mathrm{f}}_{i,\uparrow} - \hat{n}^{\mathrm{f}}_{i,\downarrow} + \hat{n}^{\mathrm{f}}_{i,\uparrow}\,\hat{n}^{\mathrm{f}}_{i,\downarrow})$. However, $\hat{P}'_i$ is unity if on the i site there are no f electrons, and is zero if on site i there is at least one f electron. As a consequence, $\hat{P}'$ gives a sum of non-negative numbers, so it is a positive semidefinite operator. Furthermore, $\hat{P}'$ gives its minimum eigenvalue (i.e. zero) for a wavefunction that contains at least one f electron on every site of the lattice. Let us consider for this reason

$$F_i^{(3)} = \sum_{\sigma} a_\sigma\, f^{\dagger}_{i,\sigma} \tag{10}$$

where the a_σ are constants to be determined. Now $|\psi_{\mathrm{g}}\rangle$ introduces three electrons L times within the system, so the solution (two bands are present) is indeed for three-quarters filling. Because of equation (10) the three electrons per lattice site are distributed in the ground state in such a way that on every site we have always at least one f electron present. As a consequence, $\hat{H} = \hat{H}_0 + \hat{U} = [\hat{H}_0 + U\,\sum_{i,\sigma} \hat{n}^{\mathrm{f}}_{i,\sigma} - U\,L] + U\,\hat{P}'$ has the ground-state wavefunction given

by equations (9), (10), provided the energy of the f level in $\hat{H}_0$ is renormalized as $E'_{\rm f} = E_{\rm f} + U$ and $E_{\rm g}$ is shifted down with $U\,L$. We underline that in the $U \to 0$ limit the described $|\psi_{\rm g}\rangle$ becomes only a small contribution from the linear combination of wavefunctions that build up the ground state. The physical properties of the interacting ground state are present only at $U > 0$, and the $U \neq 0$ state cannot be perturbatively obtained from the $U = 0$ case. This confirms the general belief that a non-Fermi-liquid emergence has to be a clear non-perturbative effect [20].

For $U > 0$, instead of $\hat{H}_0$ we have $\hat{H} = -\hat{B} + \eta\,\hat{N} - U\,L + U\,\hat{P}'$ and equations (2)–(7) all remain valid, excepting equation (3), which becomes $E'_{\rm f} = E_{\rm f} + U = |\alpha|^2 + |\beta|^2 - |\gamma|^2 - |\delta|^2$. Because equations (4)–(7) remain the same, the $\alpha,\ \beta,\ \gamma,\ \delta,\ \eta,\ m$ values remain unaltered (together with $e_1,\ e_2$ in equation (12) below); we obtain $\hat{H}'_0 = E^U_{\rm g} + \hat{P}_+$, where $E^U_{\rm g} = L\,\eta\left(1 - 2\,|m|^2\right) - U\,L$. The term $\hat{P}_+ = \hat{P} + U\,\hat{P}'$ is a positive semidefinite operator, whose minimum eigenvalue is given by $|\psi_{\rm g}\rangle$ since $\hat{P}_+\,|\psi_{\rm g}\rangle = 0$. The conditions that enable the construction of the solution for $\hat{H}$ change from equation (8) to

$$E'_{\rm f} = E_{\rm f} + U = \eta\,(1 - |m|^2) \tag{11}$$

η and V_1^2 remaining as given in equation (8) and $U > 0$. We denote the manifold defined by equation (11) by[2] D_p.

In fact $|\psi_{\rm g}\rangle$ can be written in k space as well. Fourier transforming and taking $e_1 = \alpha^* + \beta^* \exp({\rm i}k)$, $e_2 = \alpha^* m + \beta^* m^* \exp({\rm i}k)$, $F_i^{(\alpha)} = \sum_k {\rm e}^{{\rm i}kr_i} F_k^{(\alpha)}$, $D_{k,\sigma} = e_1\,c^\dagger_{k,\sigma} + e_2\,f^\dagger_{k,\sigma}$, $F_k^{(3)} = \sum_\sigma a_\sigma\,f^\dagger_{k,\sigma}$, with $F_k^{(\alpha)}\,F_k^{(\alpha)} = 0$, $F_k^{(\alpha)}\,F_{k'}^{(\alpha')} = -F_{k'}^{(\alpha')}\,F_k^{(\alpha)}$ and $M = \sum_P (-1)^p \exp[{\rm i}\,(r_1\,k_{i_1} + r_2\,k_{i_2} + \cdots + r_L\,k_{i_L})]$ where $\sum_P$ denotes a sum over all permutations of $(1,\,2,\ldots,\,L)$ to $(i_1,\,i_2,\ldots,\,i_L)$ and p represents the number of pair permutations in a given P, we find $|\psi_{\rm g}\rangle = M^3 \prod_k \left(\prod_{\alpha=1}^3 F_k^{(\alpha)}\right)|0\rangle$. With $A = M^3$ we finally obtain $|\psi_{\rm g}\rangle = A \prod_k d_k\,|0\rangle$, where

$$d_k = e_1^2\,c^\dagger_{k,\uparrow}\,c^\dagger_{k,\downarrow}\,F_k^{(3)} + e_1\,e_2\,f^\dagger_{k,\downarrow}\,f^\dagger_{k,\uparrow} \sum_\sigma a_\sigma\,c^\dagger_{k,\sigma} \tag{12}$$

and $\langle\psi_{\rm g}|\psi_{\rm g}\rangle = |A|^2 \prod_k |e_3|^2 \left[|e_1|^4 + |e_1|^2\,|e_2|^2\right]$. The e_3 constant coefficient is chosen to preserve the normalization to unity[3] and $|e_3|^2 = \sum_\sigma |a_\sigma|^2$.

To obtain an insight into the physical behaviour of the system in the ground state, let us first diagonalize $\hat{H}_0$ in momentum space. Denoting $H_k^{\rm h} = (V_k\,c^\dagger_{k,\sigma}\,f_{k,\sigma} + {\rm h.c.})$, in the thermodynamic limit we find

$$\frac{\hat{H}_0}{L} = \sum_\sigma \int_0^{2\pi} \frac{{\rm d}k}{2\pi} \left[\sum_{\rm b=c,f} \epsilon_k^{\rm b}\,b^\dagger_{k,\sigma}\,b_{k,\sigma} + H_k^{\rm h}\right] \tag{13}$$

where $\epsilon_k^{\rm c} = 2\,t_{\rm c}\cos k$, $\epsilon_k^{\rm f} = E_{\rm f} + 2\,t_{\rm f}\cos k$, $V_k = 2\,V_1\sin k + {\rm i}\,V_0$. Introducing the row vector $W_k^\dagger = (c^\dagger_{k,\sigma},\ f^\dagger_{k,\sigma})$ and the (2×2) matrix $\tilde{R}$ with components $R_{(1,1)} = \epsilon_k^{\rm c}$, $R_{(1,2)} = V_k$, $R_{(2,1)} = V_k^*$, $R_{(2,2)} = \epsilon_k^{\rm f}$, we may write the integrand in equation (13) as $W_k^\dagger\,\tilde{R}\,W_k$. The diagonalization in k space reduces to the secular equation written for $\tilde{R}$. Two bands arise, that via equation (8) become

$$\begin{aligned} E_k^{(1)} &= 2\sqrt{\bar{v}^2 + t_{\rm c}^2} > 0 \\ E_k^{(2)} &= 2\,\frac{t_{\rm f}}{t_{\rm c}}\sqrt{\bar{v}^2 + t_{\rm c}^2} + 2\,(t_{\rm f} + t_{\rm c})\cos k \end{aligned} \tag{14}$$

[2] D_p is continuous, infinite and crosses the parameter space from the low-U up to $U \to \infty$.

[3] In the text we considered $a = a_\uparrow$ a constant. However, in equation (10) the a_σ can even be local. Using the notation $|\psi_{\rm g}(a)\rangle$ for equation (12), considering $\{a_k\}$ arbitrary k-dependent sets, the complete ground state is $|\psi\rangle = \sum_{\{a_k\}} \alpha(\{a_k\})|\psi_{\rm g}(\{a_k\})\rangle$ with $\langle\psi|\psi\rangle = 1$. This reproduces equation (16), $E^U_{\rm g}$, $n_k = 3$ and main properties reported.

where we have as presented in equation (8), $\mathrm{sign}(t_\mathrm{f}) = -\mathrm{sign}(t_\mathrm{c})$. The system is at three-quarters filling, so the lower band $E_k^{(2)}$ is completely filled, and the upper band $E_k^{(1)}$, which is completely flat, is half filled. There is no hybridization between these two bands and, taking into account their filling, the ground state energy E_g is re-obtained. In the presence of $U > 0$, in the ground state, because $\hat{P}' |\psi_\mathrm{g}\rangle = 0$, the effective Hamiltonian is in fact $\hat{H}_0'/L$, which differs from $\hat{H}_0/L$ in that it has a renormalized $E_\mathrm{f}' = E_\mathrm{f} + U$, and its energy scale is shifted down with U. Effectuating the calculations for the band structure as presented for equation (14) but using instead of E_f the E_f' value, we re-obtain (shifted down with U) for the ground state the structure presented in equation (14). So we have a well defined Fermi energy, positioned at $E_k^{(1)} =$ constant, the Fermi momentum being undefinable. The system has a natural built-in gap (the minimum and nonzero distance between the $E^{(1)}$ and $E^{(2)}$) that will be visible in the physical properties at[4] $T \neq 0$.

To understand the physical behaviour of the system all ground state expectation values at $U > 0$ relevant for our study can be expressed from equation (12). Introducing $Z_k^{-1} = (|e_1|^4 + |e_1|^2\,|e_2|^2)\,|e_3|^2$, we have

$$\begin{aligned}
\langle f_{k,\sigma}^\dagger\, f_{k,\sigma}\rangle &= \left[|a_\sigma|^2\,|e_1|^4 + |e_3|^2\,|e_2|^2\,|e_1|^2\right]Z_k \\
\langle c_{k,\sigma}^\dagger\, c_{k,\sigma}\rangle &= \left[|a_\sigma|^2\,|e_1|^2\,|e_2|^2 + |e_3|^2\,|e_1|^4\right]Z_k \\
\langle c_{k,\sigma}^\dagger\, f_{k,\sigma}\rangle &= \left[|a_{-\sigma}|^2\,|e_1|^2\,e_1^*\,e_2\right]Z_k.
\end{aligned} \tag{15}$$

It is seen that $n_k = \sum_\sigma \left(\langle c_{k,\sigma}^\dagger\, c_{k,\sigma}\rangle + \langle f_{k,\sigma}^\dagger\, f_{k,\sigma}\rangle\right) = 3$ i.e. the total momentum distribution function is uniform in k space, so the system is a non-Fermi liquid. From equation (15) all individual contributions in n_k can be expressed. We obtain functions of k that are continuous together with their derivatives of any order in the whole momentum space. For example, $n_{k,\uparrow}^\mathrm{c} = \langle c_{k,\uparrow}^\dagger\, c_{k,\uparrow}\rangle = 1 - A\,(\eta + 2\,t_\mathrm{c}\cos k)(\bar{\eta} + 2\,t_\mathrm{c}\cos k)^{-1}$, where the constants are $A = |a_\downarrow|^2\,|m|^2/[|e_3|^2\,(|m|^2 - 1)]$ and $\bar{\eta} = \eta\,(|m|^2 + 1)/(|m|^2 - 1) > 2t_\mathrm{c}$. The movement of particles is allowed by $|\psi_\mathrm{g}\rangle$, that requires 'at least one f electron on every site', i.e. allows nonzero hopping matrix elements. Indeed, starting from equation (15) the ground-state expectation values of all contributions to $\hat{H}$ can be expressed. With $I/L = \int_0^{2\pi} \mathrm{d}k\,[2\,\pi\,(|e_1|^2 + |e_2|^2)]^{-1} > 0$, we find

$$\begin{aligned}
&\frac{\langle\hat{T}_\mathrm{c}\rangle}{A_1} = 2\,\eta\,|m|^2 \qquad \frac{\langle\hat{T}_\mathrm{f}\rangle}{A_1} = 2\,\eta\,|m|^4 \qquad \frac{\langle\hat{E}_\mathrm{f}\rangle}{A_2} = E_\mathrm{f} \\
&\langle\hat{U}\rangle = U\,(A_2 - L) \qquad \langle\hat{V}_0\rangle = -2\,V_0^2\,I \\
&\langle\hat{V}_1\rangle = -\frac{2\,\eta\,|m|^2\,(1 + |m|^2)\,L}{(1 - |m|^2)^2} + A_3
\end{aligned} \tag{16}$$

where $A_1 = [L - \eta\,(1 + |m|^2)\,I]/(1 - |m|^2)^2$, $A_2 = [(1 - 2\,|m|^2)\,L + 2\,\eta\,|m|^2\,I]/(1 - |m|^2)$ and $A_3 = \{2\,\eta^2\,[(1 + |m|^2)/(1 - |m|^2)]^2 - 8\,V_1^2\}\,I$. Summing up all contributions in equation (16) we re-obtain exactly the ground-state energy E_g^U. We observe from equation (16) that E_g^U cannot be expressed as a sum of on-site contributions. With $|\bar{m}|^2 = |m|^2\,(1 + |m|^2)/(1 - |m|^2)^2$, $J = \eta\,(1 + |m|^2)\,I - L$, $\langle\hat{H}_\mathrm{loc}\rangle = \langle\hat{E}_\mathrm{f}\rangle + \langle\,\hat{U}\,\rangle + \langle\hat{V}_0\rangle$, we have

$$\begin{aligned}
&\langle\hat{E}_\mathrm{f}\rangle + \langle\hat{U}\rangle + \langle\hat{H}_\mathrm{h}\rangle = E_\mathrm{g}^U + 2\,\eta\,|\bar{m}|^2\,J > E_\mathrm{g}^U \\
&\langle\hat{H}_\mathrm{loc}\rangle = E_\mathrm{g}^U + 8\,V_1^2\,I > E_\mathrm{g}^U.
\end{aligned} \tag{17}$$

Here $J = L\,[1 - r^{-2}]^{-1/2} - L > 0$ and $r = (\eta/2\,t_\mathrm{c})\,[(|m|^2 + 1)/(|m|^2 - 1)]$. From equation (17) it can be seen that the system is not localized because $\langle\hat{H}_\mathrm{loc}\rangle > E_\mathrm{g}^U$, i.e. the sum

[4] Energy is absorbed by the lower band during excitations.

of the ground-state expectation values of all on-site (localized) terms from the Hamiltonian is greater than the ground-state energy itself. On the other hand, from equation (16) $\sum_{\mathrm{b=c,f}} \langle \hat{T}_{\mathrm{b}} \rangle = -2\,\eta\,|\bar{m}|^2\,J < 0$. Furthermore, as can be seen from equations (16) and (17) adding the non-local part of the hybridization to the kinetic energy contributions $\langle \hat{T}_{\mathrm{c}} \rangle + \langle \hat{T}_{\mathrm{f}} \rangle$ we obtain also a negative number[5] $\langle \hat{H}_{\mathrm{mov}} \rangle = \langle \hat{T}_{\mathrm{c}} \rangle + \langle \hat{T}_{\mathrm{f}} \rangle + \langle \hat{V}_1 \rangle < 0$. So taken together, the sum of all ground-state expectation values connected to the movement of particles within the system is negative. As a consequence, similar to the 2D case with flat upper band and imaginary m parameter [14], in the situation described here, the system is maintaining its itinerant character instead of becoming insulating, because the $\langle \hat{H}_{\mathrm{mov}} \rangle$ (i.e. the movement of particles within the system) decreases the total energy. As a consequence, the system is in a normal phase, but without having any non-regularity in n_k at any point in k space (even in its derivatives with respect to k). From equation (10), the coefficient $a_\uparrow$ remains undetermined and the ground state has a large spin degeneracy, so it is paramagnetic (see also footnote 3).

From the point of view of the described properties is important to analyse the system also around D_p. Concerning these aspects, first of all it can be seen that the presented state emerges also in the presence of doping. Taking $N = 3L + n_r$, where $1 \leqslant n_r < L$ represents the additional electrons introduced into the system above three-quarters filling, we define $\hat{F}^{(4)} = \sum_{\{k\}} A_{\{k\}} \prod_k^{n_r} f_k^{(4)}$ with $f_k^{(4)} = \epsilon_\uparrow (f_{k,\uparrow}^+ + \mathrm{e}^{\mathrm{i}\phi_{\mathrm{c}}} c_{k,\uparrow}^+) + \epsilon_\downarrow \mathrm{e}^{\mathrm{i}\phi_\downarrow} (f_{k,\downarrow}^+ + \mathrm{e}^{\mathrm{i}\phi_{\mathrm{c}}} c_{k,\downarrow}^+)$, where $A_{\{k\}}$ represents the coefficient of a given $\{k\}$ combination of n_r different and ordered k_i taken from the L possible k-values. The ground-state wavefunction becomes under doping $|\psi_{\mathrm{gd}}\rangle = \prod_i (\prod_{\alpha=1}^3 F_i^{(\alpha)}) F^{(4)} |0\rangle$, the character of the phase described remaining unchanged since the first three operatorial components from $|\psi_{\mathrm{gd}}\rangle$ remain as in equation (9). As can be seen, the state we are describing persists also above half-filling concentration of particles within the upper band. Concerning the N dependence, we further mention that the system is a Fermi liquid for $N < 2L$.

Taking into account that the flat-band feature is intimately connected only to the presence of the $\hat{B}$ term in the Hamiltonian ($\hat{B}$ collecting all k dependences), we can maintain the described flat-band features together with the non-Fermi-liquid properties taking into consideration some local or global contributions to $\hat{H}$ that keep the system itinerant (otherwise the system becomes insulating). For example, we can leave the D_p manifold taking into account arbitrary and independent deviations δE_{f} and δU from the E_{f} and U values that satisfies equation (11). Denoting by $\delta X = \delta E_{\mathrm{f}} + \delta U$, considering $U' = U + \delta U > 0$, the new Hamiltonian becomes $\hat{H}' = \hat{H} + \delta X \sum_{i,\sigma} \hat{n}_{i,\sigma}^{\mathrm{f}}$. Adding and subtracting $\delta X \hat{N}_{\mathrm{c}}$, a new positive semidefinite operator emerges (we consider here $\delta X > 0$), $\hat{P}_X = \delta X \sum_{i,\sigma} (1 - c_{i,\sigma}^+ c_{i,\sigma})$, and we have $\hat{H}' = (\hat{P}_+ + \hat{P}_X) + E_{\mathrm{g}}'$, where, also taking doping into account, $E_{\mathrm{g}}'/L = \eta[1 - 2|m|^2 + (n_r/L)(1 + \delta X/\eta)] - U'$. The new ground-state wavefunction is obtained as $P_{\mathrm{c}} |\psi_{\mathrm{gd}}'\rangle$, where in $|\psi_{\mathrm{gd}}'\rangle$ the $F^{(4)}$ term introduces only f electrons, and $\hat{P}_{\mathrm{c}} = \prod_{i,\sigma} \hat{n}_{i,\sigma}^{\mathrm{c}}$ maximize the number of c electrons[6]. Under these conditions, at $n_r = 0$, the obtained ground-state wavefunction is a Mott insulator since it sets rigorously three electrons on every site of the lattice (a similar result is obtained at $\delta X = n_r = 0$ and real hybridization coupling constants). As a consequence, the described non-Fermi-liquid state emerges in parameter space in the close vicinity of an insulating phase. In the presence of doping, at $\delta X > 0$ the on-site expectation value $\langle \hat{H}_{\mathrm{loc}} \rangle = (\eta - |m|^2 + \delta X)(L + n_r) - U'L > E_{\mathrm{g}}'$, the system is itinerant, and since the flat-band feature is present, it is again non-Fermi liquid.

To further emphasize the itinerant character of the ground state described here we are

[5] This is because $\langle \hat{H}_{\mathrm{loc}} \rangle + \langle \hat{H}_{\mathrm{mov}} \rangle = E_{\mathrm{g}}^U$, and $\langle \hat{H}_{\mathrm{loc}} \rangle > E_{\mathrm{g}}^U$.

[6] $|\psi_{\mathrm{gd}}'\rangle$ is the ground state if after the normalization to unity the thermodynamic limit is taken. However, there are special cases (for example $t_{\mathrm{c}} > 0$, $V_0 = 0$, L odd) for which it represents the ground state even at finite L.

presenting a localized ground state obtained at three-quarters filling in another region of the phase diagram, but the same model, corresponding also to a completely flat upper band [21]

$$|\psi_1\rangle = \prod_i [(\hat{c}^\dagger_{i,\uparrow} + m\hat{f}^\dagger_{i,\uparrow})(\hat{c}^\dagger_{i,\downarrow} + m\hat{f}^\dagger_{i,\downarrow})(\alpha_i \hat{c}^\dagger_{i,\uparrow} + \beta_i \hat{c}^\dagger_{i,\downarrow} + \gamma_i \hat{f}^\dagger_{i,\uparrow} + \delta_i \hat{f}^\dagger_{i,\downarrow})]|0\rangle \tag{18}$$

where $\alpha_i, \beta_i, \gamma_i, \delta_i$ are constant numbers. This ground state contains the same number of particles (i.e. three) on every lattice site and gives [21]

$$\langle \hat{T}_c \rangle = \langle \hat{T}_f \rangle = \langle \hat{V}_1 \rangle = 0 \tag{19}$$

$$\langle \hat{H}_{loc} \rangle = E^U_{g,l} \tag{20}$$

where, for the localized case, the ground-state energy per site [21] is $E^U_{g,l}/L = -(V_o V_1/t_f)(1 - 2t_f^2/V_1^2) - U$. A comparison of the two ground states from equations (9) and (18) shows that $|\psi_1\rangle$ sets rigorously three electrons on each site of the lattice, while $|\psi_g\rangle$, besides sites with three electrons, contains sites with four and two electrons as well. As a consequence, any hopping-like Hamiltonian term (such as $\hat{T}_c$, $\hat{T}_f$ or the 'band-change hopping' $\hat{V}_1$) applied to $|\psi_1\rangle$ gives a state orthogonal to $|\psi_1\rangle$, from where equation (19) arises. This is no longer true in the case of $|\psi_g\rangle$, from where the main difference between the two ground states emerges. Starting from the point of view of the Kubo formula, we mention that we have (see e.g. [22]) $1/m^* \sim -\langle \hat{H}_{kin} \rangle$ and the real part of the conductivity in $\omega \to 0$ limit $\sigma'(\omega) \sim 1/m^*$, where m^* is the effective mass, and $\hat{H}_{kin}$ is the global kinetic energy (in our case built up from T_c, T_f and V_1, which give contributions to the current operator). As a consequence, we have a localized state only if $m^* = \infty$, i.e. in the $|\psi_1\rangle$ case where equation (19) or (20) holds.

In conclusion, in this Letter a new non-Fermi-liquid normal phase has been presented, which is not intimately connected to one spatial dimension.

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